SOLUTION.

CALCULUS II **QUIZ V** MATH-UA.0122-007

Write your solutions in steps.

1. (3 points) Find the centre of mass of the triangle with vertices at $(-1,0),(1,0),(0,1)$.
2. (4 points) Solve the differential equation:

$$y' = 2x^2 y$$

with initial value $x=1, y=1$.

3. (3 points) Determine whether the sequence $\{\frac{5^{n+3}}{6^n}\}$ converges or diverges.

1. The triangle is the region bounded by $f(x) = 1-|x|$ and x-axis

$\int_{-1}^{1} f(x)dx$ = area of triangle $= 1$

By Symmetric Principle, the centre of mass lies on y-axis. say it's $(0, y)$. Then

$$y = \frac{\int_{-1}^{1} \frac{f(x)^2}{2} dx}{\int_{-1}^{1} f(x)dx} = \int_{-1}^{1} \frac{f(x)^2}{2} dx$$

$$= 2\int_0^1 \frac{f(x)^2}{2}dx$$

because $f(x)$ is an even function

$$= 2\int_0^1 \frac{(1-x)^2}{2}dx$$

$$= \frac{1}{3}$$

So centre of mass is $(0, \frac{1}{3})$

2. $y' = 2x^2 y$.

$$\frac{dy}{y} = 2x^2 dx$$

$$\int \frac{1}{y}dy = \int 2x^2 dx$$

$$\ln|y| = \frac{2}{3}x^3 + C$$

$$|y| = e^{\frac{2}{3}x^3 + C}$$

$x=1,\ y=1>0$, so $1 = e^{\frac{2}{3}+C} \Rightarrow C = -\frac{2}{3}$

we get $y = e^{\frac{2}{3}x^3 - \frac{2}{3}}$

3. $$\lim_{n\to+\infty} \frac{5^{n+3}}{6^n} = \lim_{n\to\infty} \left(\frac{5}{6}\right)^n \times 5^3$$

$$= 125 \lim_{n\to\infty} \left(\frac{5}{6}\right)^n$$

$$= 125 \times 0$$

$$= 0.$$

So it converges.